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Solving Absolute Value Equations

Absolute Value Expressions The **absolute value** of a number is its distance from 0 on a number line. The symbol $|x|$ is used to represent the absolute value of a number x.

Absolute Value	• **Words** For any real number a, if a is positive or zero, the absolute value of a is a. If a is negative, the absolute value of a is the opposite of a. • **Symbols** For any real number a, $\|a\| = a$, if $a \geq 0$, and $\|a\| = -a$, if $a < 0$.

Example 1: Evaluate $|-4| - |-2x|$ if $x = 6$.

$|-4| - |-2x| = |-4| - |-2 \bullet 6|$
$= |-4| - |-12|$
$= 4 - 12$
$= -8$

Example 2: Evaluate $|2x - 3y|$ if $x = -4$ and $y = 3$.

$|2x - 3y| = |2(-4) - 3(3)|$
$= |-8 - 9|$
$= |-17|$
$= 17$

Exercises

Evaluate each expression if $w = -4$, $x = 2$, $y = \frac{1}{2}$, and $z = -6$.

1. $|2x - 8|$

2. $|6 + z| - |-7|$

3. $5 + |w + z|$

4. $|x + 5| - |2w|$

5. $|x| - |y| - |z|$

6. $|7 - x| + |3x|$

7. $|w - 4x|$

8. $|wz| - |xy|$

9. $|z| - 3\,|5yz|$

10. $5\,|w| + 2\,|z - 2y|$

11. $|z| - 4\,|2z + y|$

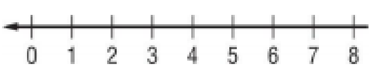
12. $10 - |xw|$

Solving Inequalities

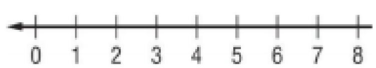
Multi–Step Inequalities An inequality is a statement that involves placing the inequality sign between two expressions. In order to solve the inequality, you need to find the set of all the values of the variable that makes the inequality true.

Solve each inequality. Then graph the solution set on a number line.

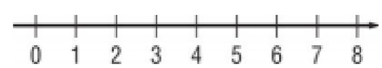
1. $c \geq \frac{c+4}{3}$



2. $r + 7 < 3(2r - 6)$



3. $3h < \frac{2h + 26}{5}$



4. Jim makes \$8.50 an hour. Each week, 26% of his total pay is deducted for taxes. How many hours does Jim have to work if he wants his take–home pay to be at least \$300 per week? Write and solve an inequality for this situation.

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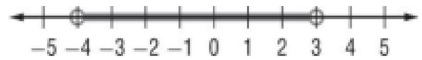
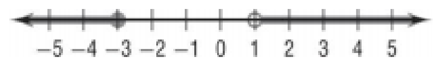
Solving Compound and Absolute Value Inequalities

Compound Inequalities A compound inequality consists of two inequalities joined by the word *and* or the word *or*. To solve a compound inequality, you must solve each part separately.

***And* Compound Inequalities**	The graph is the intersection of solution sets of two inequalities.	Example: $x > -4$ and $x < 3$ 
***Or* Compound Inequalities**	The graph is the union of solution sets of two inequalities.	Example: $x \le -3$ or $x > 1$ 

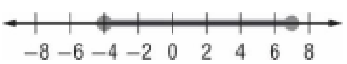
Example 1: Solve $-3 \le 2x + 5 \le 19$.
Graph the solution set on a number line.

$-3 \le 2x + 5$ and $2x + 5 \le 19$

$-8 \le 2x$ $\qquad 2x \le 14$

$-4 \le x$ $\qquad x \le 7$

$-4 \le x \le 7$



Example 2: Solve $3y - 2 \ge 7$ or $2y - 1 \le -9$.
Graph the solution set on a number line.

$3y - 2 \ge 7$ or $2y - 1 \le -9$

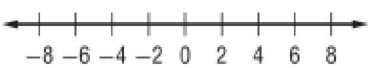
$3y \ge 9$ or $2y \le -8$

$y \ge 3$ or $y \le -4$

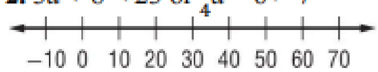
Exercises

Solve each inequality. Graph the solution set on a number line.

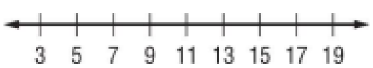
1. $-10 < 3x + 2 \le 14$



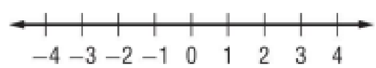
2. $3a + 8 < 23$ or $\frac{1}{4}a - 6 > 7$



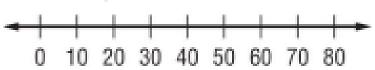
3. $18 < 4x - 10 < 50$



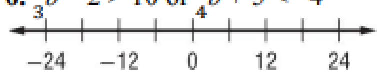
4. $5k + 2 < -13$ or $8k - 1 > 19$



5. $100 \le 5y - 45 \le 225$



6. $\frac{2}{3}b - 2 > 10$ or $\frac{3}{4}b + 5 < -4$



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Solving Compound and Absolute Value Inequalities

Absolute Value Inequalities Use the definition of absolute value to rewrite an absolute value inequality as a compound inequality.

For all real numbers a and b, $b > 0$, the following statements are true.
1. If $\|a\| < b$, then $-b < a < b$.
2. If $\|a\| > b$, then $a > b$ or $a < -b$.
These statements are also true for ≤ and ≥, respectively.

Example 1: Solve $|x + 2| > 4$. Graph the solution set on a number line.

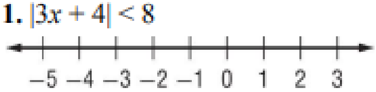
By statement 2 above, if $|x + 2| > 4$, then $x + 2 > 4$ or $x + 2 < -4$. Subtracting 2 from both sides of each inequality gives $x > 2$ or $x < -6$.

Example 2: Solve $|2x - 1| < 5$. Graph the solution set on a number line.

By statement 1 above, if $|2x - 1| < 5$, then $-5 < 2x - 1 < 5$. Adding 1 to all three parts of the inequality gives $-4 < 2x < 6$.
Dividing by 2 gives $-2 < x < 3$.

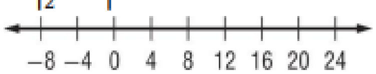
Exercises
Solve each inequality. Graph the solution set on a number line.

1. $|3x + 4| < 8$

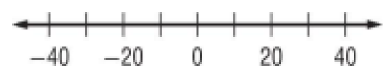


2. $|4k| + 1 > 27$

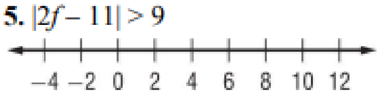
3. $\left|\frac{c}{2} - 3\right| \leq 5$



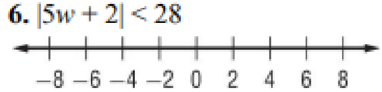
4. $|a + 9| \geq 30$



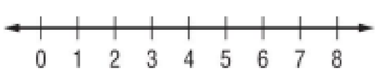
5. $|2f - 11| > 9$



6. $|5w + 2| < 28$



7. $|10 - 2k| < 2$



8. $\left|\frac{x}{2} - 5\right| + 2 > 10$

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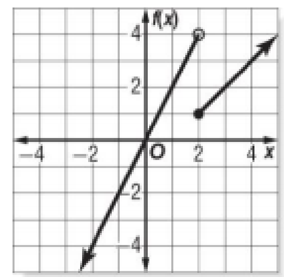
Special Functions

Piecewise-Defined Functions A **piecewise-defined function** is written using two or more expressions. Its graph is often disjointed.

Example: Graph $f(x) = \begin{cases} 2x \text{ if } x < 2 \\ x - 1 \text{ if } x \geq 2. \end{cases}$

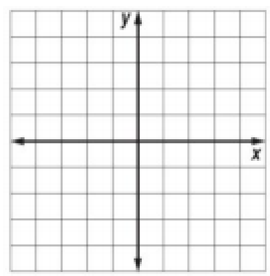
First, graph the linear function $f(x) = 2x$ for $x < 2$. Since 2 does not satisfy this inequality, stop with a circle at (2, 4). Next, graph the linear function $f(x) = x - 1$ for $x \geq 2$. Since 2 does satisfy this inequality, begin with a dot at (2, 1).



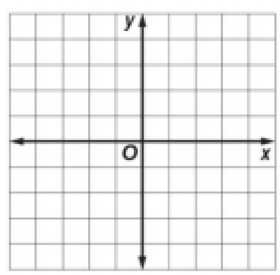
Exercises

Graph each function. Identify the domain and range.

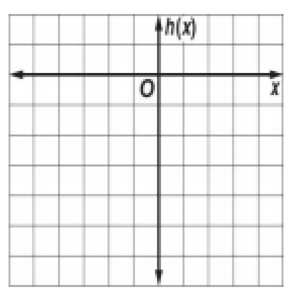
1. $f(x) = \begin{cases} x + 2 \text{ if } x < 0 \\ 2x + 5 \text{ if } 0 \leq x \leq 2 \\ -x + 1 \text{ if } x > 2 \end{cases}$



2. $f(x) = \begin{cases} -x - 4 \text{ if } x < -7 \\ 5x - 1 \text{ if } -7 \leq x \leq 0 \\ 2x + 1 \text{ if } x > 0 \end{cases}$



3. $h(x) = \begin{cases} \frac{x}{3} \text{ if } x \leq 0 \\ 2x - 6 \text{ if } 0 < x < 2 \\ 1 \text{ if } x \geq 2 \end{cases}$



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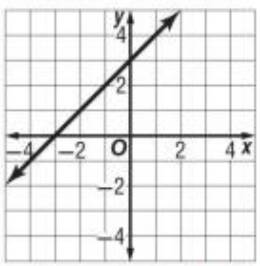
Parent Graphs The **parent graph,** which is the graph of the **parent function,** is the simplest of the graphs in a family. Each graph in a **family of graphs** has similar characteristics.

Name	Characteristics	Parent Function
Constant Function	Straight horizontal line	$y = a$, where a is a real number
Linear Function	Straight diagonal line	Identify Function $y = x$
Absolute Value Function	Diagonal lines shaped like a V	$y = \|x\|$
Quadratic Function	Curved like a parabola	$y = x^2$

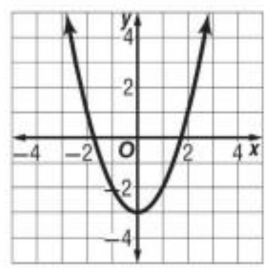
Example: Identify the type of function represented by each graph.

a.



The graph is a diagonal line. The graph represents a linear function.

b.

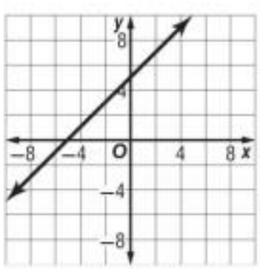


The graph is a parabolic curve. The graph represents a quadratic function.

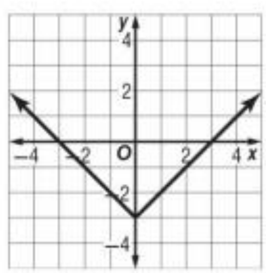
Exercises

Identify the type of function represented by each graph.

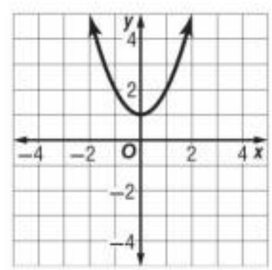
1.



2.



3.



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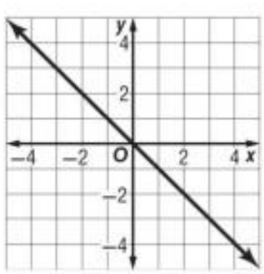

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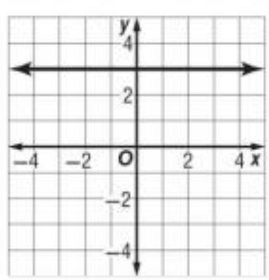

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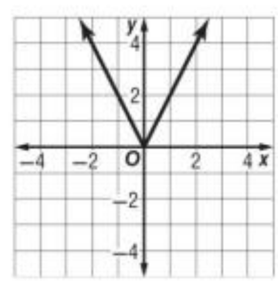

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Solving Systems of Equations

Solve Systems Graphically A system of equations is two or more equations with the same variables. You can solve a system of linear equations by using a table or by graphing the equations on the same coordinate plane. If the lines intersect, the solution is that intersection point. The following chart summarizes the possibilities for graphs of two linear equations in two variables.

Graphs of Equations	Slopes of Lines	Classification of System	Number of Solutions
Lines intersect	Different slopes	Consistent and independent	One
Lines coincide (same line)	Same slope, same *y*-intercept	Consistent and dependent	Infinitely many
Lines are parallel	Same slope, different *y*-intercepts	Inconsistent	None

Example: Graph the system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

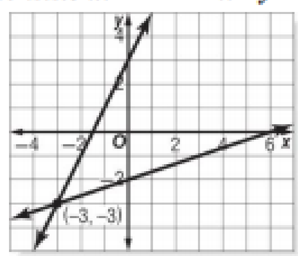
$x - 3y = 6$
$2x - y = -3$

Write each equation in slope-intercept form.

$x - 3y = 6 \quad \rightarrow \quad y = \frac{1}{3}x - 2$

$2x - y = -3 \quad \rightarrow \quad y = 2x + 3$

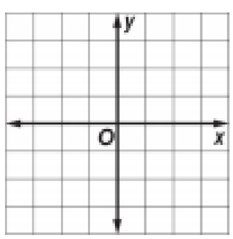
The graphs intersect at (–3, –3). Since there is one solution, the system is consistent and independent.



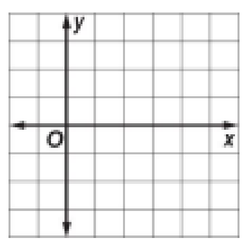
Exercises

Graph each system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

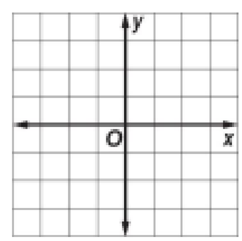
1. $3x + y = -2$
$6x + 2y = 10$



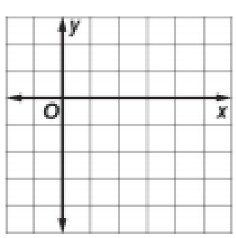
2. $x + 2y = 5$
$3x - 15 = -6y$



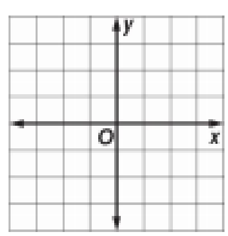
3. $2x - 3y = 0$
$4x - 6y = 3$



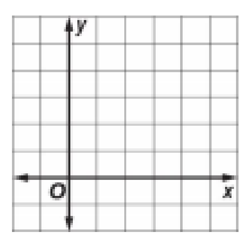
4. $2x - y = 3$
$x + 2y = 4$



5. $4x + y = -2$
$2x + \frac{y}{2} = -1$



6. $3x - y = 2$
$x + y = 6$



Name...

Solve Systems Algebraically To solve a system of linear equations by **substitution**, first solve for one variable in terms of the other in one of the equations. Then substitute this expression into the other equation and simplify. To solve a system of linear equations by **elimination**, add or subtract the equations to eliminate one of the variables.

Example 1: Use substitution to solve the system of equations. $2x - y = 9$
$x + 3y = -6$

Solve the first equation for y in terms of x.

$2x - y = 9$	First equation
$-y = -2x + 9$	Subtract $2x$ from both sides.
$y = 2x - 9$	Multiply both sides by -1.

Substitute the expression $2x - 9$ for y into the second equation and solve for x.

$x + 3y = -6$	Second equation
$x + 3(2x - 9) = -6$	Substitute $2x - 9$ for y.
$x + 6x - 27 = -6$	Distributive Property
$7x - 27 = -6$	Simplify.
$7x = 21$	Add 27 to each side.
$x = 3$	Divide each side by 7.

Now, substitute the value 3 for x in either original equation and solve for y.

$2x - y = 9$	First equation
$2(3) - y = 9$	Replace x with 3.
$6 - y = 9$	Simplify.
$-y = 3$	Subtract 6 from each side.
$y = -3$	Multiply each side by -1.

The solution of the system is $(3, -3)$.

Example 2: Use the elimination method to solve the system of equations.
$3x - 2y = 4$
$5x + 3y = -25$

Multiply the first equation by 3 and the second equation by 2. Then add the equations to eliminate the y variable.

$3x - 2y = 4$	**Multiply by 3.**	$9x - 6y = 12$
$5x + 3y = -25$	**Multiply by 2.**	$\underline{10x + 6y = -50}$
		$19x = -38$
		$x = -2$

Replace x with -2 and solve for y.

$3x - 2y = 4$
$3(-2) - 2y = 4$
$-6 - 2y = 4$
$-2y = 10$
$y = -5$

The solution is $(-2, -5)$

Exercises

Solve each system of equations.

1. $3x + y = 7$
$4x + 2y = 16$

2. $2x + y = 5$
$3x - 3y = 3$

3. $2x + 3y = -3$
$x + 2y = 2$

4. $2x - y = 7$
$6x - 3y = 14$

5. $4x - y = 6$
$2x - \frac{y}{2} = 4$

6. $5x + 2y = 12$
$-6x - 2y = -14$

7. $2x + y = 8$
$3x + \frac{3}{2}y = 12$

8. $7x + 2y = -1$
$4x - 3y = -13$

9. $3x + 8y = -6$
$x - y = 9$